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CHAPTER

3 Polynomials and Exponents

If Diane makes \$13 per hour and works 40 hours per week, she will earn more than one million dollars over the next 40 years. How much money will Diane earn? Perform the computations using scientific notation.

3–1 ■ Exponents—I

Exponential form

In chapter 1, we discussed the idea of exponents as related to real numbers. Since variables are symbols for real numbers, let us now apply the idea of exponents to variables. The expression x^4 is called the **exponential form** of the product

$$x \cdot x \cdot x \cdot x$$

We call x the **base** and 4 the **exponent.**

$$\text{Exponential form} \rightarrow \overset{\text{Exponent}}{\underset{\text{Base}}{x^4}} = \underbrace{x \cdot x \cdot x \cdot x}_{\text{4 factors of } x} \leftarrow \text{Expanded form}$$

Definition of exponents

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdot \cdot a}_{n \text{ factors of } a}, \text{ where } n \text{ is a positive integer.}$$

Concept

The exponent tells us how many times the base is used as a factor in an indicated product.

Note An exponent acts only on the symbol immediately to its left. That is, in ab^4 the exponent 4 applies only to b, whereas $(ab)^4$ means the exponent applies to both a and b.

Example 3–1 A

Write in exponential form.

1. $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$
2. $a \cdot a \cdot a = a^3$
3. $(a + b)(a + b)(a + b) = (a + b)^3$

Note In example 3, $(a + b)$ is the base.

4. $(-3)(-3)(-3)(-3) = (-3)^4$
5. $-(3 \cdot 3 \cdot 3 \cdot 3) = -3^4$

Note Examples 4 and 5 review the ideas from section 1–6 on exponents related to real numbers. Recall that $(-3)^4 = 81$, whereas $-3^4 = -81$.

▶ ***Quick check*** Write $y \cdot y \cdot y \cdot y$ in exponential form. ■

Example 3–1 B

Write as an indicated product.

1. $b^4 = b \cdot b \cdot b \cdot b$
2. $5^3 = 5 \cdot 5 \cdot 5$
3. $(x - y)^4 = (x - y)(x - y)(x - y)(x - y)$
4. $(-2)^2 = (-2)(-2)$
5. $-2^2 = -(2 \cdot 2)$

▶ ***Quick check*** Write c^5 as an indicated product. ■

Multiplication of like bases

Consider the indicated product of $x^2 \cdot x^3$. If we rewrite x^2 and x^3 by using the definition of exponents, we have

$$x^2 \cdot x^3 = \overbrace{x \cdot x}^{x^2} \cdot \overbrace{x \cdot x \cdot x}^{x^3}$$

and again using the definition of exponents, this becomes

$$x^2 \cdot x^3 = \overbrace{x \cdot x \cdot x \cdot x \cdot x}^{\text{5 factors}} = x^5$$

This leads us to the observation that

Add exponents

$$x^2 x^3 = x^{2+3} = x^5$$

Multiply like bases

Base remains unchanged

Thus we have the following **product property of exponents.**

Product property of exponents

$$a^m \cdot a^n = a^{m+n}$$

Concept

When multiplying like bases, add their exponents.

Note The base stays the same throughout the process. It is by adding the exponents that the multiplication is carried out.

Example 3–1 C

Find the product.

1. $x^3 \cdot x^5 = x^{3+5} = x^8$

2. $3^2 \cdot 3^4 = 3^{2+4} = 3^6 = 729$

Note A common error in multiplying $3^2 \cdot 3^4$ is to multiply the bases $3 \cdot 3 = 9$ and add the exponents, getting the incorrect answer of 9^6. The correct way is to say $3^2 \cdot 3^4 = 3^6$, not 9^6.

3. $y^2 \cdot y^3 \cdot y^4 = y^{2+3+4} = y^9$

4. $a^2 \cdot a \cdot a^3 = a^{2+1+3} = a^6$

Note The variable a means the same as a^1. Likewise, 3 means the same as 3^1. If there is no exponent written with a numeral or a variable, the exponent is understood to be 1.

5. $(a+b)^3(a+b)^4 = (a+b)^{3+4} = (a+b)^7$

6. $(-2)^3(-2)^2 = (-2)^{3+2} = (-2)^5 = -32$

▶ ***Quick check*** Find the product. $x^4 \cdot x^5$ ■

Group of factors to a power property of exponents

Several additional properties of exponents can be derived using the definition of exponents and the commutative and associative properties of multiplication. Observe the following:

$$(xy)^3 = \overbrace{xy \cdot xy \cdot xy}^{\text{3 factors of } xy}$$
$$= \overbrace{x \cdot x \cdot x}^{\text{3 factors of } x} \cdot \overbrace{y \cdot y \cdot y}^{\text{3 factors of } y}$$
$$= x^3y^3$$

This leads us to the following property of exponents.

Group of factors to a power property of exponents

$$(ab)^n = a^nb^n$$

Concept

When a group of *factors* is raised to a power, raise each of the factors in the group to this power.

Example 3–1 D

Simplify.

1. $(ab)^4 = a^4b^4$ Both a and b are raised to the 4th power

Groups of factors to a power		Raise each factor to the power		Standard form
2. $(2ab)^3$	$=$	$2^3a^3b^3$	$=$	$8a^3b^3$

Note In example 2, the number 2 is a factor in the group. Therefore it is also raised to the indicated power.

3. $(3 \cdot 4)^3 = 3^3 \cdot 4^3 = 27 \cdot 64 = 1{,}728$ $(3 \cdot 4)^3$ also is $(12)^3 = 1{,}728$ ■

Note The quantity $(a + b)^3 \neq a^3 + b^3$ because a and b are *terms,* not factors as the property specified. If we consider $(a + b)$ to be a single factor, then by the definition of exponents we have

$$(a + b)^3 = (a + b)(a + b)(a + b)$$

We will see the method of multiplying this later in this chapter.

Power of a power

Consider the expression $(x^4)^3$. Applying the definition of exponents and the product property of exponents, we have

$$(x^4)^3 = \overbrace{x^4 \cdot x^4 \cdot x^4}^{\text{3 factors of } x^4} = \overbrace{x^{4+4+4}}^{\text{Add the exponents}} = x^{12}$$

In chapter 1, we reviewed the idea that multiplication is repeated addition of the same number. Therefore adding the exponent 4 three times is the same as $4 \cdot 3$. Thus

Power of a power   Multiply exponents

$$(x^4)^3 = x^{4 \cdot 3} = x^{12}$$

Therefore we have the following property of exponents.

Power of a power property of exponents

$$(a^m)^n = a^{m \cdot n}$$

Concept
A power of a power is found by multiplying the exponents.

■ Example 3–1 E

Simplify.

1. $(y^3)^2 = y^{3 \cdot 2} = y^6$

2. $(4^2)^5 = 4^{2 \cdot 5} = 4^{10} = 1{,}048{,}576$

3. $(x^5)^4 = x^{5 \cdot 4} = x^{20}$

▶ ***Quick check*** Simplify. $(a^4)^3$ ■

Products of monomials

To multiply the monomials

$$3x^2 \cdot 5x$$

we apply the commutative and associative properties of multiplication along with the properties of exponents. We then write this expression as a product of the numerical coefficients times the product of the variables. That is,

$$3x^2 \cdot 5x = (3 \cdot 5)(x^2 \cdot x) = 15x^3$$

To find the product of

$$5a \cdot 4b$$

we apply the same properties to get

$$5a \cdot 4b = (5 \cdot 4)(a \cdot b) = 20ab$$

Note It is a good procedure to write the variable factors of any term in alphabetical order. This makes identifying like terms much simpler. For example, $3a^2c^3b$ and $4bc^3a^2$ are like terms, but recognizing that fact would have been easier if they had been written as $3a^2bc^3$ and $4a^2bc^3$.

Example 3–1 F

Perform the indicated multiplication.

1. $4x \cdot 3xy = (4 \cdot 3) \cdot (x \cdot x) \cdot y = 12x^2y$
2. $8a^3 \cdot 4a^3 \cdot 3a = (8 \cdot 4 \cdot 3) \cdot (a^3 \cdot a^3 \cdot a) = 96a^7$
3. $(-2a^2) \cdot (3ab) = (-2 \cdot 3) \cdot (a^2 \cdot a) \cdot b = -6a^3b$

Note The product of a^3 and b can *only* be written as a^3b since a and b are not like bases.

4. $(5x^2y^3z)(4x^3yz^4) = (5 \cdot 4)(x^2x^3)(y^3y)(zz^4) = 20x^5y^4z^5$ ■

Problem solving

The following problems require us to write algebraic expressions involving the use of exponents.

Example 3–1 G

Write an algebraic expression for each of the following verbal statements.

1. The volume of a cube is found by using the length of the edge, e, as a factor 3 times. Write an expression for the volume of a cube.

   We write e as a factor 3 times as $e \cdot e \cdot e = e^3$. Then the volume, V, of a cube is given by

   $V = e^3$.

2. Write an expression for 5 less than the square of a number.

   Let n represent the number, then the square of the number is given as n^2, and since "less than" means to subtract, the expression is given by

   $n^2 - 5$. ■

Mastery points

Can you

- Write a product in exponential form?
- Use the product property of exponents?
- Raise a group of factors to a power?
- Raise a power to a power?
- Multiply monomials?

Exercise 3–1

Write the following expressions in exponential form. See example 3–1 A.

Example $y \cdot y \cdot y \cdot y$

Solution $= y^4$ *y to the fourth power*

1. $aaaaa$
2. $bbbb$
3. $(-2)(-2)(-2)(-2)$
4. $-(2 \cdot 2 \cdot 2 \cdot 2)$
5. $xxxxxx$
6. $(2a)(2a)(2a)$
7. $(xy)(xy)(xy)(xy)$
8. $(a + b)(a + b)$
9. $(x - y)(x - y)(x - y)$
10. $(2a - b)(2a - b)(2a - b)$

Write as an indicated product (expanded form). See example 3–1 B.

Example c^5

Solution $= c \cdot c \cdot c \cdot c \cdot c$ *c written as a factor 5 times*

11. x^4
12. y^5
13. $(-2)^3$
14. -2^4
15. 5^3
16. $(5x)^3$
17. $(4y)^4$
18. $(a + b)^3$
19. $(x - y)^2$
20. $(2x + y)^3$

Simplify by using the properties of exponents. See examples 3–1 C, D, E, and F.

Examples $x^4 \cdot x^5$

Solutions $= x^{4+5}$ Like bases

$= x^9$ Add exponents

$(a^4)^3$

$= a^{4 \cdot 3}$ Power of a power

$= a^{12}$ Multiply exponents

21. $x^4 \cdot x^7$
22. $a^5 \cdot a^5$
23. $R^2 \cdot R$
24. $a \cdot a^4$
25. $a^2 \cdot a^3 \cdot a^4$
26. $x^5 \cdot x \cdot x^3$
27. $5^2 \cdot 5^3$
28. $6 \cdot 6^3$
29. $4 \cdot 4^2 \cdot 4^4$
30. $(a + b)^2(a + b)^5$
31. $(x - 2y)^4(x - 2y)^6$
32. $(3a + b)^2(3a + b)^3$
33. $(a - b)^4(a - b)^7$
34. $(ab)^5$
35. $(xy)^4$
36. $(2abc)^3$
37. $(4xyz)^3$
38. $(a^2)^4$
39. $(x^5)^3$
40. $(y^2)^2$
41. $(b^5)^5$
42. $(c^9)^3$
43. $(2xy^2)(3x^3y)$
44. $(4x^2y^3)(5xy^4)$
45. $(a^2b^3)(a^5b^2)$
46. $(x^2y^2)(x^4y^3)$
47. $(6x^3)(5x^2)$
48. $(4a)(3a^4)$
49. $(2a^3b^4c)(6a^4b^3)$
50. $(5xy)(xy)$
51. $(3a^2b)(4a^3b^2)$
52. $(a^3b^4)(5a^2b^5)$
53. $(-2a^2b)(3ab^4)$
54. $(-5x^2y^5)(-2x^2y)$
55. The formula for finding the volume of a cube is $V = e^3$, where V represents volume in some cubic unit of measure and e represents the length of the edge of the cube. Write an expression for the volume in expanded form, and then determine the number of cubic units in the figure for each of the following values of e: (a) $e = 5$, (b) $e = 4$, (c) $e = 6$.

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Using exponents, write an expression for each of the following verbal statements. See example 3–1 G.

56. The area, A, of a square is found by using the length of the side, s, as a factor twice. Write an expression for the area of a square.

57. The distance, s, a falling object will fall in time, t, seconds is found by multiplying $\frac{1}{2}$ times the gravity, g, times the square of t. Write an expression for the distance the object will fall.

58. The area of a circle is found by multiplying the constant π times the length of the radius, r, used as a factor 2 times. Write an expression for the area of a circle.

59. The volume, V, of a sphere is found by multiplying $\frac{4}{3}\pi$ times the radius, r, used as a factor 3 times. Write an expression for the volume of a sphere.

$\boxed{\textbf{60.}}$ Johnny is n years old. His mother says that she is 6 years more than the cube of Johnny's age. Write an expression for his mother's age.

61. Jane is m years old. Her father is 8 years less than Jane's age used as a factor 4 times. Write an expression for her father's age.

62. Write an expression for 2 times the square of t.

63. Write an expression for twice the square of x less the cube of y.

64. A number can be written in the form a times 10 used as a factor 8 times, where a is a number between 1 and 10. Write an expression for the number in terms of a.

65. Write an expression for the quotient of the cube of p divided by the square of q.

Review exercises

Perform the indicated addition and subtraction. See section 2–3.

1. $2a + 3a + 4a$ **2.** $5x + x + 2x$ **3.** $3ab - 2ab + 5ab$

4. $9xy + 4xy - 6xy$ **5.** $4a^2 + 3a^2 - 2a + 7a$ **6.** $6x^2 + 3x - x^2 + 2x$

7. $2x^2y - x^2y + 3xy^2 + 4xy^2$ **8.** $5ab^2 + 3a^2b - 2ab^2 - a^2b$

3–2 ■ Products of algebraic expressions

Product of a monomial and a multinomial

To multiply a monomial and a multinomial (a polynomial of more than one term), we use the distributive property. For example, to multiply

$$3x^2y(x^2 + xy + y^2)$$

we multiply each term in the trinomial by the monomial $3x^2y$ to get

$$(3x^2y \cdot x^2) + (3x^2y \cdot xy) + (3x^2y \cdot y^2)$$

which yields

$$3x^2y(x^2 + xy + y^2) = 3x^4y + 3x^3y^2 + 3x^2y^3$$

In each indicated product, note that we multiplied like bases by using the properties of exponents. For example, in the first term,

$$3x^2y \cdot x^2 = 3 \cdot (x^2 \cdot x^2) \cdot y = 3 \cdot x^{2+2} \cdot y = 3x^4y$$

■ Example 3–2 A

Perform the indicated multiplication.

1. $5y(2y + 3) = 5y \cdot 2y + 5y \cdot 3$ Distribute $5y$ times each term in the parentheses
$= 10y^2 + 15y$ Multiply monomials

2. $x^3(x^2 + xy - y^2) = x^3 \cdot x^2 + x^3 \cdot xy - x^3 \cdot y^2$
$= x^5 + x^4y - x^3y^2$

Note In example 2, when we multiplied x^3 times the third term of the trinomial, y^2, the subtraction sign remained, giving $-x^3y^2$.

3. $-5a^3(a^2 + 2ab - b^3) = -5a^3 \cdot a^2 - 5a^3 \cdot 2ab + 5a^3 \cdot b^3$
$= -5a^5 - 10a^4b + 5a^3b^3$

4. $4x^2y(2x^3 - 3x^2y^2 + y^4) = 4x^2y \cdot 2x^3 - 4x^2y \cdot 3x^2y^2 + 4x^2y \cdot y^4$
$= 8x^5y - 12x^4y^3 + 4x^2y^5$

▶ *Quick check* Perform the indicated multiplication. $3ab^2(2a - 3b)$ ■

Product of two multinomials

The product of two multinomials requires the use of the distributive property several times. That is, in the product

$$(x + 2y)(x + y)$$

we consider $(x + 2y)$ a single number and apply the distributive property.

$$(x + 2y)(x + y) = (x + 2y) \cdot x + (x + 2y) \cdot y$$

We now apply the distributive property again.

$$(x + 2y) \cdot x + (x + 2y) \cdot y = x \cdot x + 2y \cdot x + x \cdot y + 2y \cdot y$$
$$= x^2 + 2xy + xy + 2y^2$$

The last step in the problem is to combine like terms, if there are any.

$$x^2 + (2xy + xy) + 2y^2 = x^2 + 3xy + 2y^2$$

Notice that in this product, each term of the first factor is multiplied by each term of the second factor. We can generalize our procedure as follows:

Multiplying two multinomials

When we are multiplying two multinomials, we multiply each term in the first multinomial by each term in the second multinomial. We then combine like terms.

■ Example 3–2 B

Perform the indicated multiplication and simplify.

1. $(a + 3)(a - 4) = \overset{1}{a \cdot a} - \overset{2}{a \cdot 4} + \overset{3}{3 \cdot a} - \overset{4}{3 \cdot 4}$ Distribute multiplication
$= a^2 - 4a + 3a - 12$ Multiply monomials
$= a^2 - a - 12$ Combine like terms

Note We have drawn arrows to indicate the multiplication that is being carried out. This should be a convenient way for us to indicate the multiplication to be performed.

$$\textbf{2. } (2x + 3)(5x - 2) = \overset{F}{10x^2} - \overset{O}{4x} + \overset{I}{15x} - \overset{L}{6}$$
$$= 10x^2 + 11x - 6$$

Distribute and multiply
Combine like terms

Note A word that is useful for remembering the multiplication to be performed when multiplying two binomials is **FOIL.** Foil is an abbreviation signifying **F**irst times first, **O**uter times outer, **I**nner times inner, and **L**ast times last.

$$\textbf{3. } (a + b)(a + 2b) = a^2 + 2ab + ab + 2b^2$$
$$= a^2 + 3ab + 2b^2$$

Distribute and multiply
Combine like terms

▶ ***Quick check*** Perform the indicated multiplication and simplify. $(2x + y)(x - 3y)$ ■

Special products

Three special products appear so often that the form of the answers can be written without computation. Consider the product

$$(x + 6)^2 = (x + 6)(x + 6)$$

which becomes

$$x^2 + 6x + 6x + 36$$

When we combine the second and third terms, we get

$$x^2 + 12x + 36$$

This is called the **square of a binomial** or a **perfect square trinomial** and has certain characteristics. Inspection shows us that in

$$(x + 6)^2 = x^2 + 12x + 36$$

the three terms of the product can be obtained in the following manner:

The square of a binomial

1. The first term of the product is the *square of the first term* of the binomial $[(x)^2 = x^2]$.
2. The second term of the product is *two times the product of the two terms of the binomial* $[2(x \cdot 6) = 12x]$.
3. The third term of the product is the *square of the second term* of the binomial $[(6)^2 = 36]$.

If we apply this to

$$(x - 7)^2$$
$$= [x + (-7)]^2$$

we get

$$= x^2 + [2 \cdot x \cdot (-7)] + (-7)^2$$

and so

$$(x - 7)^2 = x^2 + (-14x) + 49$$
$$= x^2 - 14x + 49$$

In general, for real numbers a and b,

$$(a + b)^2 = a^2 + 2ab + b^2$$

and

$$(a - b)^2 = a^2 - 2ab + b^2$$

Note $(a + b)^2 = a^2 + 2ab + b^2$, *not* $a^2 + b^2$. This is a common error. *The square of a binomial is always a trinomial.*

■ Example 3–2 C

Perform the indicated multiplication and simplify.

1. $(2x + 3)^2 = (2x)^2 + (2 \cdot 2x \cdot 3) + (3)^2$ — Apply special products property
   $= 4x^2 + 12x + 9$ — Multiply monomials
2. $(5a - 4b)^2 = (5a)^2 - [2 \cdot 5a \cdot (4b)] + (4b)^2$ — Special products property
   $= 25a^2 - [40ab] + 16b^2$ — Multipy monomials
   $= 25a^2 - 40ab + 16b^2$ — Standard form ■

The third special product is obtained by multiplying the sum and the difference of the same two terms. Consider the following:

$$\begin{aligned}(x + 3)(x - 3) &= x^2 - 3x + 3x - 9\\ &= x^2 - 9\end{aligned}$$

Special characteristics are evident in this product also.

The difference of two squares

For real numbers a and b,

$$(a + b)(a - b) = a^2 - b^2$$

Concept

1. The product is obtained by first squaring the first term of the factors, and then
2. subtracting the square of the second term of the factors.

■ Example 3–2 D

Perform the indicated multiplication and simplify.

1. $(x + 7)(x - 7) = (x)^2 - (7)^2 = x^2 - 49$
2. $(a + 2b)(a - 2b) = (a)^2 - (2b)^2 = a^2 - 4b^2$
3. $(3x - 2y)(3x + 2y) = (3x)^2 - (2y)^2 = 9x^2 - 4y^2$ ■

In all the examples that we have looked at, whether they were special products or not, a single rule is sufficient.

When multiplying two multinomials together, we multiply each of the terms in the first multinomial times each of the terms in the second multinomial and then combine like terms.

■ Example 3–2 E

Perform the indicated multiplication and simplify.

1. $(3x - y)(2x + 3y) = 6x^2 + 9xy - 2xy - 3y^2$ Distribute multiplication
   $= 6x^2 + 7xy - 3y^2$ Combine like terms

2. $(a - 2)(2a^2 + 3a + 2) = 2a^3 + 3a^2 + 2a - 4a^2 - 6a - 4$ Distribute multiplication
   $= 2a^3 - a^2 - 4a - 4$ Combine like terms

Note Although there are three terms in the second parentheses, we still follow the procedure of every term in the first parentheses times every term in the second parentheses.

3. $(x - y)(x^2 + 3xy - y^2)$ Distribute multiplication
   $= x^3 + 3x^2y - xy^2 - x^2y - 3xy^2 + y^3$ Combine like terms
   $= x^3 + 2x^2y - 4xy^2 + y^3$

4. $(a + 6)(a - 2)(a - 1)$. When there are three multinomials to be multiplied, we apply the associative property to multiply two of them together first and take that product times the third.

$$
\begin{aligned}
[(a + 6)(a - 2)](a - 1) &= [a^2 - 2a + 6a - 12](a - 1) \\
&= [a^2 + 4a - 12](a - 1) \\
&= a^3 - a^2 + 4a^2 - 4a - 12a + 12 \\
&= a^3 + 3a^2 - 16a + 12
\end{aligned}
$$
■

Mastery points

Can you

- Multiply a monomial and a multinomial?
- Multiply multinomials?
- Find the special products of the square of a binomial or the difference of two squares?

Exercise 3–2

Perform the indicated multiplication and simplify. See examples 3–2 A, B, C, D, and E.

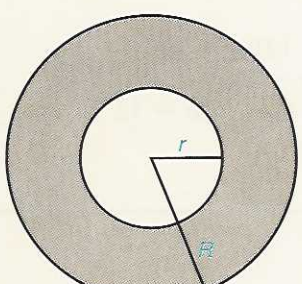
Examples $3ab^2(2a - 3b)$

Solutions $= 3ab^2 \cdot 2a - 3ab^2 \cdot 3b$ Distributive property
$= 6a^2b^2 - 9ab^3$ Multiply monomials

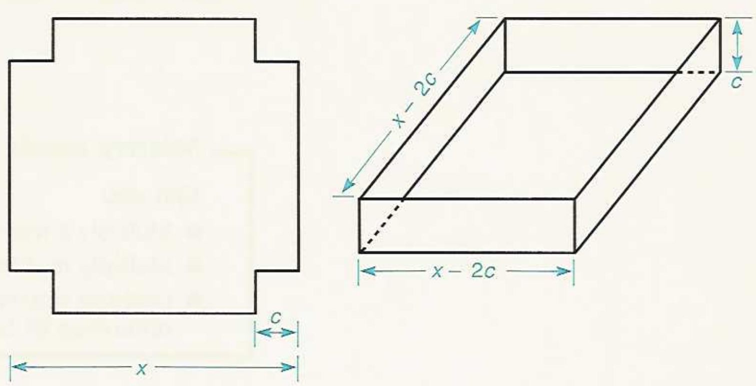
$(2x + y)(x - 3y)$

$= 2x \cdot x - 2x \cdot 3y + y \cdot x - y \cdot 3y$ Distributive property
$= 2x^2 - 6xy + xy - 3y^2$ Multiply monomials
$= 2x^2 - 5xy - 3y^2$ Combine like terms

1. $2ab(a^2 - bc + c^2)$
2. $6x(4y + 7z)$
3. $3a(5b^2 - 7c^2)$
4. $-ab(a^4 - a^2b^2 - b^4)$
5. $-5ab^2(3a^2 - ab + 4b^2)$
6. $6x^2(4x^2 - 2x + 3)$
7. $3ab(a^2 - 2ab - b^2)$
8. $(2x)(x - y + 5)(5y)$
9. $(3a)(2a - b)(2b^2)$
10. $(x^2y)(x^2 + y^2)(xy^2)$
11. $(x + 3)(x + 4)$
12. $(a + 5)(a - 3)$
13. $(y - 9)(y - 4)$
14. $(z + 7)(z - 11)$
15. $(a + 1)(a + 1)$
16. $(b - 1)(b - 1)$
17. $(R - 3)^2$
18. $(R + 2)(R - 2)$
19. $(a + 3)(a - 3)$
20. $(3x + 2)(x - 4)$
21. $(3a - 5)(2a - 7)$
22. $(3 - 2y)(2 - y)$
23. $(7 + 2x)(2x - 7)$
24. $(4r + 3)(r - 12)$
25. $(3k + w)(k - 6w)$
26. $(a - 6bc)(5a + 4bc)$
27. $(a + 6b)^2$
28. $(2a + 3b)^2$
29. $(2a + 3b)(2a - 3b)$
30. $(4x - y)(4x + y)$
31. $(a + 4b)(a^2 - 2ab + b^2)$
32. $(x - 2y)(2x^2 - 3xy + y^2)$
33. $(x + 4)(6x^2 - 3x + 7)$
34. $(x - y)(x^2 - 2xy + y^2)$
35. $(x^2 - 2x - 3)(x^2 + x + 4)$
36. $(a^2 - 3a + 6)(a^2 + 2a - 5)$
37. $(a - 6)(a - 2)(a + 1)$
38. $(2b - 1)(b + 2)(2b + 1)$
39. $(a - b)(a + b)(2a - 3b)$
40. $(a + b)^3$
41. $(a - b)^3$
42. $(2a + b)^3$
43. $(a - 2b)^3$
44. The area of the shaded region between the two circles is $\pi(R + r)(R - r)$. Perform the indicated multiplication.



45. When squares of c units on a side are cut from the corners of a square sheet of metal x units on a side, and the metal sheet is then folded up into a tray, the volume is $c(x - 2c)(x - 2c)$. Perform the indicated multiplication.



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Review exercises

Perform the indicated addition or subtraction. See sections 1–4 and 1–5.

1. $(-3) + (-2)$ **2.** $8 - (-4)$ **3.** $(-7) - (-10)$ **4.** $2 + (-5) + (-6)$

Simplify by using the properties of exponents. See section 3–1.

5. $x^4 \cdot x^8$ **6.** $(a^3)^5$ **7.** $(3ab)^3$ **8.** $(2x)^3$

3–3 ■ Exponents–II

Fraction to a power property of exponents

In section 3–1, we learned several useful properties of exponents. Now we shall learn several more.

Our next property of exponents can be derived from the definition of exponents. Consider the expression $\left(\frac{a}{b}\right)^3$.

$$\left(\frac{a}{b}\right)^3 = \underbrace{\frac{a}{b}\cdot\frac{a}{b}\cdot\frac{a}{b}}_{\text{3 factors of } \frac{a}{b}} = \frac{\overbrace{a\cdot a\cdot a}^{\text{3 factors of } a}}{\underbrace{b\cdot b\cdot b}_{\text{3 factors of } b}} = \frac{a^3}{b^3}$$

Thus

$$\underset{\text{Fraction raised to a power}}{\left(\frac{a}{b}\right)^3} = \frac{a^3}{b^3} \quad \begin{array}{l}\leftarrow \text{Numerator raised to the power}\\ \leftarrow \text{Denominator raised to the power}\end{array}$$

Fraction to a power property of exponents

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n},\ b \neq 0$$

Concept

Whenever a fraction is raised to a power, the numerator and the denominator are *both* raised to that power.

■ Example 3–3 A

Perform the indicated operations and simplify.

1. $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$

2. $\left(\frac{a}{b}\right)^5 = \frac{a^5}{b^5}$

3. $\left(\frac{2a}{b}\right)^3 = \frac{(2a)^3}{b^3} = \frac{2^3a^3}{b^3} = \frac{8a^3}{b^3}$ ■

Division of expressions with like bases

Consider the expression

$$\frac{x^6}{x^2}$$

We can use the definition of exponents to write the fraction as

$$\frac{x^6}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$$

We reduce the fraction as follows:

$$\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{x \cdot x \cdot x \cdot x}{1} = \frac{x^4}{1} = x^4$$

In our example, we reduced by two factors of x, leaving $6 - 2 = 4$ factors of x in the numerator. Therefore

$$\frac{x^6}{x^2} = x^{6-2} = x^4$$

Thus we have the following property of exponents.

Quotient property of exponents

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n},\ a \neq 0$$

Concept

To divide quantities having *like* bases, subtract the exponent of the denominator from the exponent of the numerator to get the power of the given base in the quotient.

Note If the base a is zero, $a = 0$, we have an expression that has no meaning. Therefore $a \neq 0$ indicates that we want our variables to assume no values that would cause the denominator to be zero.

Example 3–3 B

Simplify. Assume that no variable is equal to zero.

1. $\frac{x^7}{x^5} = x^{7-5} = x^2$

2. $a^{11} \div a^4 = a^{11-4} = a^7$

3. $\frac{5^4}{5} = 5^{4-1} = 5^3 = 125$

Note Remember that when we are dividing like bases, their exponents are subtracted, but *the base is not changed.*

4. $\frac{a^5 \cdot a^2}{a^4} = \frac{a^{5+2}}{a^4} = \frac{a^7}{a^4} = a^{7-4} = a^3$

5. $\frac{x^3}{y^2} = \frac{x^3}{y^2}$

Note In example 5, we cannot simplify. The bases must be the same in order to subtract exponents when we divide.

6. $\dfrac{2^5x^9y^{15}}{2^3x^5y^{12}} = 2^{5-3}x^{9-5}y^{15-12} = 2^2x^4y^3 = 4x^4y^3$

▶ ***Quick check*** Simplify. Assume that no variable is equal to zero. $a^{11} \div a^7$ ■

Negative exponents

To this point, we have considered only those problems where the exponent of the numerator is greater than the exponent of the denominator. Consider the example

$$\frac{x^2}{x^6}$$

By the definition of exponents, this becomes

$$\frac{x^2}{x^6} = \frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x}$$

and reducing the fraction,

$$\frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x \cdot x \cdot x \cdot x} = \frac{1}{x^4}$$

Again, we reduced by two factors of x, leaving $6 - 2 = 4$ factors of x in the denominator. Hence

$$\frac{x^2}{x^6} = \frac{1}{x^4}$$

However using the quotient property of exponents to carry out the division, we would have

$$\frac{x^2}{x^6} = x^{2-6} = x^{-4}$$

Since we should arrive at the same answer regardless of which procedure we use, then x^{-4} must be $\dfrac{1}{x^4}$, thus $x^{-4} = \dfrac{1}{x^4}$. This leads us to the definition of negative exponents.

Definition of negative exponents

$$a^{-n} = \frac{1}{a^n},\ a \neq 0$$

Concept

A negative exponent on any base (except zero) can be written as 1 over that base with a positive exponent.

■ ***Example 3–3 C***

Write the following problems with positive exponents. Assume that no variable is equal to zero.

1. $x^{-3} = \dfrac{1}{x^3}$ Rewritten as 1 over x to the positive 3rd

2. $a^{-9} = \dfrac{1}{a^9}$ Rewritten as 1 over a to the positive 9th

Note From the definition of negative exponents, if a **factor** is moved from either the numerator to the denominator or from the denominator to the numerator, the sign of its exponent will change. The sign of the base will not be affected by this change.

2. *Alternative procedure*

$$a^{-9} = \frac{a^{-9}}{1}$$ Rewrite as a fraction

$$= \frac{1}{a^9}$$ Sign of the exponent is changed as the **factor** is moved from the numerator to the denominator

3. $$\frac{1}{b^{-4}} = \frac{b^4}{1}$$ Sign of the exponent is changed as the **factor** is moved from the denominator to the numerator

$$= b^4$$ Standard form is to leave only positive exponents

4. $$(-3)^{-3} = \frac{1}{(-3)^3}$$ Sign of the exponent is changed as the factor is moved from the numerator to the denominator

$$= \frac{1}{-27} \text{ or } -\frac{1}{27}$$ Standard form

▶ ***Quick check*** Write b^{-2} with positive exponents. ■

Zero as an exponent

Now consider the situation involving the division of like bases that are raised to the same power.

$$\frac{x^3}{x^3}, x \neq 0$$

By the definition of exponents, we have

$$\frac{x^3}{x^3} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{1}{1} = 1$$

By the quotient property of exponents,

$$\frac{x^3}{x^3} = x^{3-3} = x^0$$

Since $\frac{x^3}{x^3} = 1$ and $\frac{x^3}{x^3} = x^0$, then x^0 must be equal to 1. This leads us to the definition of zero as an exponent.

Definition of zero as an exponent

$$a^0 = 1, a \neq 0$$

Concept

Any number other than zero raised to the zero power is equal to 1.

Example 3–3 D

Simplify. Assume that no variable is equal to zero.

1. $b^0 = 1$ **2.** $r^0 = 1$ **3.** $5^0 = 1$ **4.** $(-2)^0 = 1$

5. $(a + b)^0 = 1$ **6.** $3x^0 = 3 \cdot 1 = 3$ **7.** $(3x)^0 = 1$

Note The exponent acts only on the symbol immediately to its left. In example 6, only the x is raised to the zero power. The exponent of 3 is understood to be 1. In example 7, the parentheses indicate that both the 3 and the x are raised to the zero power.

▶ ***Quick check*** Simplify. Assume that no variable is equal to zero. C^0

Example 3–3 E

Simplify. Leave the answer with only positive exponents. Assume that no variable is equal to zero.

1.
$$\begin{aligned} \frac{x^5}{x^{11}} &= x^{5-11} && \text{Division of like bases} \\ &= x^{-6} && \text{Subtract exponents} \\ &= \frac{1}{x^6} && \text{Standard form} \end{aligned}$$

2.
$$\begin{aligned} a^{-7} \cdot a^5 &= a^{-7+5} && \text{Multiplication of like bases} \\ &= a^{-2} && \text{Add exponents} \\ &= \frac{1}{a^2} && \text{Standard form} \end{aligned}$$

3.
$$\begin{aligned} (b^{-2})^{-4} &= b^{(-2)\cdot(-4)} && \text{Power of a power} \\ &= b^8 && \text{Multiply exponents} \end{aligned}$$

4.
$$\begin{aligned} \frac{a^3b^5}{a^7b^2} &= a^{3-7}b^{5-2} && \text{Division of like bases} \\ &= a^{-4}b^3 && \text{Subtract exponents} \\ &= \frac{b^3}{a^4} && \text{Standard form} \end{aligned}$$

5.
$$\begin{aligned} \frac{a^3b^2c^4}{ab^5c^4} &= a^{3-1}b^{2-5}c^{4-4} && \text{Division of like bases} \\ &= a^2b^{-3}c^0 && \text{Subtract the exponents} \\ &= \frac{a^2 \cdot 1}{b^3} && \text{The } a\text{'s remain in the numerator, the } b\text{'s drop to the denominator, and } c^0 \text{ is 1} \\ &= \frac{a^2}{b^3} && \text{Standard form} \end{aligned}$$

6.
$$\begin{aligned} \frac{a^{-2}b^4}{a^{-5}b^6} &= a^{-2-(-5)}b^{4-6} && \text{Division of like bases} \\ &= a^3b^{-2} && \text{Subtract exponents} \\ &= \frac{a^3}{b^2} && \text{Standard form} \end{aligned}$$

▶ ***Quick check*** Simplify. Leave the answer with only positive exponents. Assume that no variable is equal to zero. $\dfrac{b^4}{b^{10}}$

Mastery points

Can you

- Raise a fraction to a power?
- Perform division on expressions having like bases?
- Perform operations involving negative exponents?
- Perform operations involving zero as an exponent?

Exercise 3–3

Write each expression with only positive exponents. Assume that no variable is equal to zero. See examples 3–3 C and D.

Examples C^0 | b^{-2}

Solutions $= 1$ By definition is equal to 1 | $= \dfrac{1}{b^2}$ Rewritten as 1 over b to the positive 2nd

1. x^0
2. $(2y)^0$
3. $\boxed{3.}$ $5a^0$
4. $7x^0$
5. $(3B)^0$
6. S^{-2}
7. R^{-5}
8. $(2x)^{-3}$
9. $(3P)^{-2}$
10. $\boxed{10.}$ $4z^{-2}$
11. $9C^{-4}$
12. $\dfrac{5}{x^{-4}}$
13. $\dfrac{1}{2y^{-3}}$
14. $\dfrac{1}{3x^{-2}}$
15. $\boxed{15.}$ $2x^{-4}y^2$
16. $x^{-2}y^4$
17. $p^0r^{-2}t^5$
18. $x^{-3}y^2z^{-4}$

Perform all indicated operations and leave your answer with only positive exponents. Assume that no variable is equal to zero. See examples 3–3 A, B, and E.

Examples $a^{11} \div a^7$ | $\dfrac{b^4}{b^{10}}$

Solutions $= a^{11-7}$ Division of like bases
$= a^4$ Subtract exponents

$= b^{4-10}$ Division of like bases
$= b^{-6}$ Subtract exponents
$= \dfrac{1}{b^6}$ Standard form

19. $\left(\dfrac{a}{b}\right)^6$
20. $\left(\dfrac{x}{y}\right)^4$
21. $\left(\dfrac{2}{3}\right)^3$
22. $\left(\dfrac{1}{2}\right)^4$
23. $\left(\dfrac{2x}{y}\right)^4$
24. $\left(\dfrac{2ab}{c}\right)^3$
25. $\left(\dfrac{3a}{b}\right)^3$
26. $x^{12} \div x^6$
27. $y^4 \div y^2$
28. $\dfrac{a^5}{a^3}$
29. $\dfrac{b^9}{b^7}$
30. $\dfrac{c^6}{c^9}$
31. $\dfrac{R^4}{R^8}$
32. $\boxed{32.}$ $\dfrac{3^4}{3^2}$
33. $\dfrac{2^5}{2^3}$
34. $\dfrac{4^2}{4^5}$
35. $\dfrac{6}{6^3}$
36. $\boxed{36.}$ $\dfrac{x^4x^3}{x^2}$
37. $\dfrac{y^5y}{y^2}$
38. $\dfrac{a^4a^2}{a^5}$
39. $\dfrac{a^4}{a^2a}$
40. $\dfrac{x^7}{x^2x^3}$
41. $\dfrac{y^3}{y^4y^5}$
42. $\dfrac{b^2}{bb^4}$
43. $\dfrac{a^7b^5}{a^4b^2}$
44. $\dfrac{x^9y^7}{x^4y}$
45. $\dfrac{2^3x^3y^7}{2xy^5}$
46. $\dfrac{3^3a^4b^5}{3^2a^2b^3}$
47. $\dfrac{3a^2b^5}{3^4a^5b^5}$
48. $\boxed{48.}$ $\dfrac{5^2a^3b}{5^3a^7b^3}$

$\boxed{49.}$ $x^{-4}x^7$ 50. $y^{-2}y^{10}$ 51. a^5a^{-11} 52. $R^{-2}R^{-5}$ 53. $x^{-2}x^4x^0$

54. $x^5x^0x^{-2}$ 55. $a^0a^{-5}a^3$ 56. $a^{-7}a^4a^0$ 57. $(-5)^{-3}$ 58. $(-2)^{-4}$

59. $\dfrac{3^{-2}}{3^{-5}}$ 60. $\dfrac{2^{-6}}{2^{-3}}$ 61. $(a^{-2})^3$ 62. $(b^4)^{-4}$ 63. $(x^5)^{-2}$

64. $(y^{-3})^4$ 65. $(a^{-2})^{-3}$ 66. $(z^{-4})^{-4}$ 67. $(x^0)^{-2}$ 68. $(a^{-3})^0$

69. $(x^{-2})^0$ 70. $(b^0)^{-4}$ 71. $\dfrac{R^2S^{-4}}{R^{-3}S^5}$ 72. $\dfrac{4y^{-3}}{4^{-1}y^2}$ $\boxed{73.}$ $\dfrac{4^{-1}a^{-2}b^3c^0}{2a^{-3}b^{-1}c^{-2}}$

Review exercises

Perform the indicated operations. See sections 1–4 to 1–8.

1. $(-4) + (-6)$ 2. $(-3)(-7)$ 3. $(-4) - (-8)$ 4. -4^2

Simplify by using the properties of exponents. See section 3–1.

5. a^3a^5 6. $(x^3)^4$ 7. xx^2x^3 8. $(2ab)^2$

3–4 ■ Exponents—III

Properties and definitions of exponents

The following is a summary of the properties and definitions of exponents that we have studied so far.

Definitions

$$a^n = \overbrace{a \cdot a \cdot a \cdots a}^{n \text{ factors}}, \text{ where } n \text{ is a positive integer}$$

$$a^{-n} = \frac{1}{a^n}, a \neq 0$$

$$a^0 = 1, a \neq 0$$

Properties

$$a^m \cdot a^n = a^{m+n}$$

$$(ab)^n = a^nb^n$$

$$(a^m)^n = a^{m \cdot n}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$$

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}, a \neq 0$$

The following examples illustrate some more problems in which more than one property of exponents is applied within the same problem.

■ Example 3–4 A

Simplify. Leave the answer with only positive exponents. Assume that no variable is equal to zero.

1. $(2a^2b^3)^3 = 2^3(a^2)^3(b^3)^3$ — Each factor in the group is raised to the 3rd power
$= 2^3a^6b^9$ — Power of a power, multiply exponents
$= 8a^6b^9$ — 8 is the standard form of 2^3

2. $(5a^4b^2)^4 = 5^4(a^4)^4(b^2)^4$ — Each factor is raised to the power
$= 5^4a^{16}b^8$ — Power of a power
$= 625a^{16}b^8$ — Standard form

3. $(3a^{-2}b^3)^{-3} = 3^{-3}(a^{-2})^{-3}(b^3)^{-3}$ — Each factor is raised to the power
$= 3^{-3}a^6b^{-9}$ — Power of a power
$= \dfrac{a^6}{3^3b^9}$ — The 3s and the b's drop to the denominator
$= \dfrac{a^6}{27b^9}$ — Standard form

4. $(-3a^2)(2ab^3)(-4a^3b^5)$
$= [(-3)(2)(-4)](a^2aa^3)(b^3b^5)$ — Multiply like bases using the commutative and associative properties
$= 24a^6b^8$ — Signed numbers and multiplication of like bases, add exponents

5. $(3a^4b)^2(3^2ab^5)^2 = 3^2(a^4)^2b^2 \cdot (3^2)^2a^2(b^5)^2$ — Group of factors to a power
$= 3^2a^8b^2 \cdot 3^4a^2b^{10}$ — Power of a power
$= (3^2 \cdot 3^4)(a^8a^2)(b^2b^{10})$ — Multiply like bases
$= 3^6a^{10}b^{12}$ — Add exponents
$= 729a^{10}b^{12}$ — Standard form

6. $\left(\dfrac{2a^2b^3}{c^5}\right)^3 = \dfrac{(2a^2b^3)^3}{(c^5)^3}$ — Both the numerator and the denominator are raised to the 3rd power
$= \dfrac{2^3(a^2)^3(b^3)^3}{(c^5)^3}$ — Each factor in the numerator is raised to the 3rd power
$= \dfrac{8a^6b^9}{c^{15}}$ — Power of a power, multiply exponents

7. $\dfrac{a^{-2}b^3}{a^{-4}b^6} = a^{(-2)-(-4)}b^{3-6}$ — Division of like bases
$= a^2b^{-3}$ — Subtract exponents
$= \dfrac{a^2}{b^3}$ — Standard form

8. $\left(\dfrac{a^{-2}b}{c^3}\right)^{-2} = \dfrac{(a^{-2}b)^{-2}}{(c^3)^{-2}}$ — Numerator and denominator are raised to the power
$= \dfrac{(a^{-2})^{-2}b^{-2}}{(c^3)^{-2}}$ — Numerator has a group of factors to a power
$= \dfrac{a^{(-2)(-2)}b^{-2}}{c^{(3)(-2)}}$ — Power of a power
$= \dfrac{a^4b^{-2}}{c^{-6}}$ — Multiply exponents
$= \dfrac{a^4c^6}{b^2}$ — Standard form, factors raised to a negative power are moved to the other side of the fraction bar

▶ ***Quick check*** Simplify. Leave the answer with only positive exponents. Assume that no variable is equal to zero. $(2a^{-2}b^3)^3$ ■

Mastery points

Can you

- Apply the definitions and properties of exponents?

Exercise 3–4

Simplify by using the properties and definitions of exponents. Leave the answer with only positive exponents. Assume that no variable is equal to zero. See example 3–4 A.

Example $(2a^{-2}b^3)^3$

Solution

$= 2^3(a^{-2})^3(b^3)^3$ Groups of factors to a power

$= 2^3a^{-6}b^9$ Power of a power

$= \dfrac{8b^9}{a^6}$ Standard form

1. $(2a^2)^3$ **2.** $(3x^4)^2$ **3.** $(2x^2y)^3$ **4.** $(4ab^3)^2$

5. $(x^4y^3z)^4$ **6.** $(2a^5b^2c)^3$ **7.** $(5a^5b^2c^4)^2$ **8.** $(a^3b^2)^3$

$\boxed{\textbf{9.}}$ $(2a^2)^{-2}$ **10.** $(5x^{-3})^{-2}$ **11.** $(4^{-1}x^2)^{-2}$ **12.** $(2a^2b^{-3})^{-2}$

13. $(3xy^{-4})^{-3}$ **14.** $(3x^{-2}y^{-3})^2$ **15.** $(x^2y^{-5}z^3)^{-2}$ **16.** $(a^{-3}b^2c^{-4})^{-3}$

17. $(3x^2)(2x^0y^2)(x^5y)$ **18.** $(a^2b)(-3a^0b^2)(a^3b)$ **19.** $(-2x^2y)(3x^3y^2)(x^5y)$

20. $(a^2bc)(-2a^2b^2c^2)(3abc)$ **21.** $(x^3yz^4)(-3xyz^2)(-2x^2yz)$ **22.** $(a^2b)(-3b^2c^2)(2a^2c^2)$

23. $\left(\dfrac{2x}{y^2}\right)^3$ **24.** $\left(\dfrac{x^2y}{z^2}\right)^3$ **25.** $\left(\dfrac{3a^2c^0}{b^3}\right)^2$ **26.** $\left(\dfrac{x^3}{y^0z^4}\right)^3$

27. $\left(\dfrac{2x^2}{y^3}\right)^3$ **28.** $\left(\dfrac{ab^2}{c^4}\right)^4$ **29.** $\left(\dfrac{2x^3y^3}{z^5}\right)^2$ **30.** $\left(\dfrac{a^5bc^4}{d^2e}\right)^5$

$\boxed{\textbf{31.}}$ $\dfrac{a^{-2}b^3}{a^3b^{-5}}$ **32.** $\dfrac{x^{-5}y^2}{x^3y^{-4}}$ **33.** $\dfrac{3R^{-1}S^{-2}}{9R^{-3}S^2}$ **34.** $\dfrac{R^2S^{-4}}{R^{-3}S^5}$

35. $\dfrac{8a^{-2}b^{-5}}{2a^{-1}b^4}$ **36.** $\dfrac{2x^{-1}y^{-2}}{3x^{-2}y^2}$ **37.** $\dfrac{6R^{-2}S^0}{2R^2S^{-3}}$ **38.** $\dfrac{2a^{-1}b^0c^2}{5a^3b^{-1}c^{-3}}$

$\boxed{\textbf{39.}}$ $(a^2b^3)^3(ab^2)^4$ **40.** $(xy^2)^3(x^2y^2)^2$ **41.** $(2a^3)^2(2a^2)^3$ **42.** $(3x^5)^3(3x^3)^2$

43. $(2x^2y)^3(2x^4y^5)^2$ **44.** $(3r^2s^4)^3(r^5s^6)^2$ **45.** $\left(\dfrac{xy^{-2}}{z^{-4}}\right)^{-1}$ **46.** $\left(\dfrac{x^{-3}y}{z^5}\right)^{-2}$

47. $\left(\dfrac{2a^{-3}}{b^5}\right)^{-2}$ **48.** $\left(\dfrac{4^{-1}a^{-2}}{b^{-5}}\right)^{-2}$ **49.** $\left(\dfrac{ab^{-2}}{c^{-1}}\right)^{-3}$ **50.** $\left(\dfrac{2^{-2}x^3}{y^{-2}}\right)^{-3}$

Review exercises

Perform the indicated multiplication. See section 1–2.

1. $(6.2) \cdot (5.7)$ **2.** $(2.8) \cdot (3.7)$ **3.** $(1.9) \cdot (8.8)$

4. $(4.2) \cdot (6.9)$ **5.** $(9.9) \cdot (1.9)$ **6.** $(7.5) \cdot (6.6)$

3–5 ■ Scientific notation

Scientific notation

An important use of integer exponents is in scientific, engineering, and technical fields where we deal with very large or very small numbers. For example, the mass of a hydrogen atom is 0.000 000 000 000 000 000 000 001 67 gram; the mass of an electron is 0.000 000 000 000 000 000 000 000 000 91 gram; the half-life of lead-204 is 14,000,000,000,000,000,000 years. To work with such numbers on the calculator, they must often be entered in scientific notation. We define the scientific notation of a positive number X to be the product

$$X = a \times 10^n$$

where $1 \leq a < 10$ and n is an integer. To achieve this form of the decimal number X, use the following steps.

Scientific notation

Step 1 Move the decimal point to a position immediately following the first nonzero digit in X.

Step 2 Count the number of places the decimal point has been moved. This is the power, n, to which 10 is raised.

Step 3 If
- a. the decimal point is moved to the *left*, n is *positive*.
- b. the decimal point is moved to the *right*, n is *negative*.
- c. the decimal point already follows the first nonzero digit, n is *zero*.

■ **Example 3–5 A**

Express the following numbers in scientific notation.

1. 250

$250 = 2.50. \times 10^2 = 2.5 \times 10^2$

2. 45,000,000

$45{,}000{,}000 = 4.5000000. \times 10^7 = 4.5 \times 10^7$

3. 5

$5 = 5 \times 10^0$

4. 0.000152

$0.000152 = 0.0001.52 \times 10^{-4} = 1.52 \times 10^{-4}$

Note To write a negative number in scientific notation, we use the same procedure as for a positive number except that a negative sign, −, is placed in front of a.

Example
To write −0.0234 in scientific notation, we proceed as follows:

$$-0.0234 = -0.02.34 \times 10^{-2} = -2.34 \times 10^{-2}$$

▶ ***Quick check*** Express the following numbers in scientific notation.
4,380 −0.00592 ■

Standard form

Sometimes it is necessary to convert a number in scientific notation to its standard form. To do this, we apply the rules in reverse.

Standard form

When the power of 10 is
1. *positive,* the decimal point is moved to the *right n* places.
2. *negative,* the decimal point is moved to the *left n* places.
3. *zero,* the decimal point is not moved.

Example 3–5 B

Express the following numbers in standard form.

1. 1.45×10^4
Since the exponent of 10 is positive 4, we move the decimal point 4 places to the *right* to get

$1.45 \times 10^4 = 1.4500. = 14{,}500$

2. 5.23×10^{-3}
The *negative* exponent, -3, tells us to move the decimal point 3 places to the *left* to get

$5.23 \times 10^{-3} = 0.005.23 = 0.00523$

Note In each example, it was necessary to insert zeros to properly locate the decimal point.

3. -4.07×10^{-2}
With a negative exponent, -2, move the decimal point 2 places to the *left* to get

$-4.07 \times 10^{-2} = -0.04.07 = -0.0407$

Note The negative sign preceding the number is carried along into the standard form.

▶ ***Quick check*** Express the following numbers in standard form.
9.98×10^{-4} $\quad -5.63 \times 10^4$

Computation using scientific notation

Scientific notation can be used to simplify numerical calculations when the numbers are very large or very small. We first change the numbers to scientific notation and use the properties of exponents to help perform the indicated operations.

Example 3–5 C

Perform the indicated operations using scientific notation.

1. $(349{,}000{,}000)(0.0816)$

$= (3.49 \times 10^8)(8.16 \times 10^{-2})$	Scientific notation
$= (3.49 \cdot 8.16) \times (10^8 \cdot 10^{-2})$	Commutative and associative properties
$= 28.4784 \times 10^6$	Multiply
$= 28{,}478{,}400$	Standard form

2. $\dfrac{(102{,}000{,}000)(0.00105)}{(1{,}190)(0.012)}$

$= \dfrac{(1.02 \times 10^8)(1.05 \times 10^{-3})}{(1.19 \times 10^3)(1.2 \times 10^{-2})}$ Scientific notation

$= \dfrac{(1.02)(1.05)10^8 \cdot 10^{-3}}{(1.19)(1.2)10^3 \cdot 10^{-2}}$ Commutative and associative properties

$= \dfrac{(1.02)(1.05)}{(1.19)(1.2)} \times 10^4$ Properties of exponents

$= 0.75 \times 10^4$ Multiplication and division

$= 7{,}500$ Standard form

Mastery points

Can you

- Express a number in scientific notation?
- Convert a number from scientific notation to standard form?
- Do computations using scientific notation?

Exercise 3–5

Express the following numbers in scientific notation. See example 3–5 A.

Examples 4,380 | −0.00592

Solutions $= 4.38 \times 10^3$ Three places to the left, exponent is 3 | $= -5.92 \times 10^{-3}$ Three places to the right, exponent is −3

1. 255	**2.** 65,000,000	**3.** 12,345	**4.** 14,800	**5.** 155,000
6. 14.36	**7.** 855.076	**8.** 1,570.7	**9.** 1,007,600	**10.** 6,000,736
11. 0.00012	**12.** 0.0863	**13.** 0.0000081	**14.** 0.0000147	**15.** 0.0007
16. 0.12079	**17.** 0.000000000094	**18.** −456	**19.** −4,500	**20.** −0.00087
21. −5,850,000	**22.** −0.0567	**23.** −45.78	**24.** −34,000,000	**25.** −0.00000002985

Convert the following numbers in scientific notation to their standard form. See example 3–5 B.

Examples 9.98×10^{-4} | -5.63×10^4

Solutions $= 0.000998$ Exponent is −4, move 4 places to the left | $= -56{,}300$ Exponent is 4, move 4 places to the right

26. 2.07×10^3	**27.** 4.99×10^7	**28.** 5.061×10^5	**29.** 7.23×10^0
30. 1.073×10^4	**31.** 4.2×10^{-3}	**32.** 7.611×10^{-7}	**33.** 1.47×10^{-6}
34. 5.0×10^{-2}	**35.** 7.89×10^{-4}	**36.** -2.3×10^5	**37.** -4.82×10^{-9}
38. -2.61×10^2	**39.** -4.92×10^{-6}	**40.** -9.3×10^8	

Express the following numbers in scientific notation or in standard form. See examples 3–5 A and B.

41. A millicron equals 0.000000001 of a meter. Write this number in scientific notation.

42. The speed of light is approximately 30,000,000,000 centimeters per second. Write this in scientific notation.

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** Undergraduate students may choose to defer repayment until six months after graduation or ceasing to be enrolled at least half time in school. Interest only and immediate repayment options also available.

*** A 0.25% interest rate reduction is available for borrowers who elect to have monthly principal and interest payments transferred electronically from a savings or checking account. The interest rate reduction will begin when automatic principal and interest payments start, and will remain in effect as long as automatic payments continue without interruption. This reduced interest rate will return to contract rate if automatic payments are cancelled, rejected or returned for any reason. Upon request, borrowers are also entitled to an additional 0.25% interest rate reduction if (1) the first 36 payments of principal and interest are paid on time, and (2) at any time prior to the 36th on time payment, the borrower who receives the monthly bill elects to have monthly principal and interest payments transferred electronically from a savings or checking account, and continues to make such automatic payments through the 36th payment. This reduced interest rate will not be returned to contract rate if, after receiving the benefit, the borrower discontinues automatic electronic payment. The lender and servicer reserve the right to modify or discontinue borrower benefit programs (other than the co-signer release benefit) at any time without notice.

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Chapter 3 summary

1. In the expression x^6, x is called the **base** and 6 the **exponent.**
2. $a^n = \overbrace{a \cdot a \cdot a \cdots a}^{n \text{ factors}}$, where n is a positive integer.
3. **Properties and definitions of exponents**
   $a^m \cdot a^n = a^{m+n}$
   $(ab)^n = a^n b^n$
   $(a^m)^n = a^{m \cdot n}$
   $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, b \neq 0$
   $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$
   $a^{-n} = \dfrac{1}{a^n}, a \neq 0$
   $a^0 = 1, a \neq 0$
4. When *multiplying* two multinomials, we multiply each term in the first multinomial by each term in the second multinomial. We then combine like terms.
5. Three **special products** are:
   $(a + b)^2 = a^2 + 2ab + b^2$
   $(a - b)^2 = a^2 - 2ab + b^2$
   $(a + b)(a - b) = a^2 - b^2$
6. The scientific notation of a positive number X is $X = a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

Chapter 3 error analysis

1. Exponents
   Example: $xy^3 = x^3y^3$
   Correct answer: $xy^3 = xy^3$
   What error was made? (*see page 127*)
2. Multiplication of like bases
   Example: $4^2 \cdot 4^3 = 16^5$
   Correct answer: $4^2 \cdot 4^3 = 4^5$
   What error was made? (*see page 129*)
3. Power of a power
   Example: $(a^3)^2 = a^5$
   Correct answer: a^6
   What error was made? (*see page 130*)
4. Multiplying unlike bases
   Example: $x^3 \cdot y = (xy)^4$
   Correct answer: $x^3 \cdot y = x^3y$
   What error was made? (*see page 131*)
5. Product of a monomial and a multinomial
   Example: $-x(x^2 - 2x + 1) = x^3 - 2x^2 + x$
   Correct answer: $-x^3 + 2x^2 - x$
   What error was made? (*see page 134*)
6. Squaring a binomial
   Example: $(4x + y)^2 = (4x)^2 + (y)^2 = 16x^2 + y^2$
   Correct answer: $16x^2 + 8xy + y^2$
   What error was made? (*see page 136*)
7. Dividing like bases
   Example: $\dfrac{x^3}{x} = x^4$
   Correct answer: x^2
   What error was made? (*see page 140*)
8. Negative exponents
   Example: $\dfrac{1}{a^{-3}} = -a^3$
   Correct answer: $\dfrac{1}{a^{-3}} = a^3$
   What error was made? (*see page 141*)
9. Zero exponent
   Example: $(x + y)^0 = x^0 + y^0 = 1 + 1 = 2$
   Correct answer: $(x + y)^0 = 1$
   What error was made? (*see page 143*)
10. Evaluate absolute value
   Example: $-|-4| = 4$
   Correct answer: $-|-4| = -4$
   What error was made? (*see page 31*)

Chapter 3 critical thinking

Given the number 52^2, determine a method by which you can square the 52 mentally.

Chapter 3 review

[3–1]

Simplify and leave the answers with only positive exponents.

1. $a^5 \cdot a^7$
2. $a \cdot a^4 \cdot a^9$
3. $4^3 \cdot 4^2$
4. $(xy)^4$
5. $(a^3)^5$
6. $(5ab^3)(4a^3b^2)$
7. $(3x^2y^3)(2xy^4)$
8. $(-5x^2)(3x^3)$
9. $(2a^2b)(3ab^4)$
10. $(5x^2y)(2x^3y^4)$
11. $(-3a^2b^3)(2a^4b^7)$

[3–2]

Perform the indicated multiplication and simplify.

12. $5x(3x - 2y)$
13. $-3a^2b(2a^2 - 3ab + 4b^2)$
14. $(5x)(3x - y)(2y^2)$
15. $(x + 3)(x - 4)$
16. $(x + 5)^2$
17. $(a - 7)(a + 7)$
18. $(5x - y)(3x + 2y)$
19. $(x - 2y)(x^2 + 3xy + y^2)$
20. $(3a - b)(a + b)(a - 2b)$
21. $(2a + b)^3$

[3–3]

Simplify and leave the answers with only positive exponents.

22. $\dfrac{b^5}{b^7}$
23. $5a^{-2}$
24. $a^{-5} \cdot a^9$
25. $\dfrac{x^3x^2}{x^8}$
26. $(3a^2b)^0$
27. $5x^{-3}y^{-2}$
28. $\dfrac{a^{-4}}{a^{-7}}$
29. $\left(\dfrac{a}{b}\right)^5$
30. $\left(\dfrac{2yz}{x}\right)^2$
31. $\dfrac{2a^2b^4}{2^3a^5b}$
32. $\dfrac{a^5b^{-2}}{a^{-4}b}$

[3–4]

Simplify and leave the answers with only positive exponents.

33. $(2a^2b^3)^3$
34. $(3^3x^4y^5)^4$
35. $(2xy^{-3})^{-2}$
36. $\dfrac{2x^{-1}y^0z^3}{4x^{-2}y^{-3}}$
37. $\dfrac{8a^{-5}b^{-4}c^0}{4a^{-7}b^2c^{-3}}$
38. $(x^5y^4)^4(2x^2y^3)^3$
39. $(2a^2b)^3(3a^4)^2$
40. $\left(\dfrac{a^3b^0}{c^4}\right)^3$
41. $\left(\dfrac{3a^3b^2}{c^5}\right)^2$
42. $\left(\dfrac{2xy^4}{z^6}\right)^5$

[3–5]

Express the following numbers in scientific notation.

43. 1,840
44. 0.00157
45. 107,000,000
46. 849,000,000,000
47. -37.5
48. -0.00543

Express the following numbers in standard form.

49. 5.04×10^5
50. 6.39×10^{-3}
51. -5.96×10^2
52. -8.86×10^{-3}
53. 7.35×10^{-7}
54. 8.12×10^8

Perform the indicated operations using scientific notation. Leave the answer in scientific notation.

55. $(456{,}000{,}000) \cdot (0.000587)$
56. $(0.0000183) \cdot (0.000846)$
57. $(756{,}000) \div (105{,}000{,}000)$
58. $(0.00525) \div (42{,}000)$

Chapter 3 cumulative test

Determine if the following statements are true or false.

[1–3] 1. $|-10| < 0$

[1–8] 2. $-3^2 = (-3)^2$

[1–3] 3. $|-3| < |-7|$

Perform the indicated operations, if possible, and simplify.

[1–7] 4. $\dfrac{(8)}{(-4)}$

[1–7] 5. $\dfrac{(-9)}{0}$

[1–5] 6. $(2) - (-6)$

[1–6] 7. $(-2)(4)(0)(-4)$

[1–8] 8. $(-2)^4$

[1–8] 9. $48 - 24 \div 8 - 3 - 2^2$

[2–3] 10. $(3x^2y - 4xy + 2xy^2) - (2x^2y - 4xy + 3x^2y^2)$

[1–6] 11. $(5)(-2)(4)$

[1–7] 12. $\dfrac{0}{-3}$

[3–2] 13. $(3x - y)^2$

[1–6] 14. $(2)(-7)(0)(3)$

[3–4] 15. $(2a^2b^5)^2$

[1–6] 16. -5^2

[1–8] 17. $10 - 10 \div 10 \cdot 10 - 10 + 10$

[3–2] 18. $(3x - 2y)(3x + 2y)$

[3–1] 19. $(3x^2y)(2xy^4)$

[1–8] 20. $2[5(7 - 4) - 6 + 4]$

[1–7] 21. $\dfrac{(-6)(-4)}{(5)(0)}$

[2–3] 22. $(3a - 2b) - [5a - (4b + 6a)]$

[3–2] 23. $(x + 1)(x^2 - x - 1)$

[3–3] 24. $x^{-3}x^5x^0$

[3–3] 25. $\dfrac{a^{-5}}{a^{-9}}$

Find the solution set.

[2–6] 26. $3x - 4 = x + 10$

[2–6] 27. $2(x - 4) + 7 = 8x - 11$

[2–6] 28. $\dfrac{2}{3}x + 4 = \dfrac{5}{6}$

Find the solution.

[2–9] 29. $8 - 3x < 9$

[2–9] 30. $6x + 5 - 4 > 2$

[2–6] 31. $3 - 2x = 6$

[2–9] 32. $-9 \le 2x + 7 \le 5$

[2–8] 33. If a number is decreased by 17 and that result is then divided by 5, the answer is 16. Find the number.

[2–8] 34. One-third of a number is 12 less than one-half of the number. Find the number.

[2–8] 35. Brenda invested part of $30,000 at 8% and the rest at 7%. If her income for one year from the 8% investment was $675 more than that from the 7% investment, how much was invested at each rate?

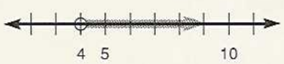
Review exercises

1. -16 **2.** 16 **3.** -16 **4.** 16 **5.** x^5 **6.** (let $x =$ the number) x^3 **7.** (let $x =$ the number) x^2 **8.** xy

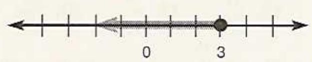
Chapter 2 review

1. 3 **2.** 1 **3.** 2 **4.** 2 **5.** polynomial **6.** polynomial **7.** polynomial **8.** not a polynomial because a variable is in the denominator **9.** $5x$ **10.** $y - 7$ **11.** $z + 4$ **12.** (let $x =$ the number) $2x + 6$ **13.** 1 **14.** -1 **15.** 72 **16.** -4 **17.** 4 **18.** 7 **19. a.** 3 **b.** $\frac{189}{4}$ **20.** 1,040 **21.** $4x^2 - 3x + 3$ **22.** $-a^2 + a + 11$ **23.** $-6a^2$ **24.** $5x^3 - 7xy^2 - 2y^3 - 4x^2$ **25.** $-11ab + 7b^2c + 11bc$ **26.** $-6y + 1$ **27.** $5ab + 3ac - 4bc$ **28.** -5 **29.** $7x + y$ **30.** $2x + 5y$ **31.** $-6x + 4y$ **32.** $9a - 10b$ **33.** true **34.** false **35.** false **36.** true **37.** $\{7\}$ **38.** $\{21\}$ **39.** $\{-11\}$ **40.** $\{-6\}$ **41.** $\{4\}$ **42.** $\{19\}$ **43.** $\{-2\}$ **44.** $\{9\}$ **45.** $\{3\}$ **46.** $\{3\}$ **47.** $\{-7\}$ **48.** $\{-7\}$ **49.** $\{12\}$ **50.** $\{14\}$ **51.** $\{15\}$ **52.** $\{21\}$ **53.** $\left\{\frac{21}{4}\right\}$ **54.** $\{-18\}$ **55.** $\left\{-\frac{9}{2}\right\}$ **56.** $\{35\}$ **57.** $\{0\}$ **58.** $\{4\}$ **59.** $\{6\}$ **60.** $\{-8\}$ **61.** $\{3\}$ **62.** $\left\{\frac{14}{3}\right\}$ **63.** $\left\{\frac{7}{3}\right\}$ **64.** $\{3\}$ **65.** $\left\{\frac{1}{2}\right\}$ **66.** $\left\{-\frac{22}{3}\right\}$ **67.** $\left\{-\frac{1}{4}\right\}$ **68.** $\left\{\frac{7}{13}\right\}$ **69.** $\left\{-\frac{5}{2}\right\}$, **70.** $\{-14\}$ **71.** $\{-24\}$ **72.** $\left\{\frac{7}{4}\right\}$ **73.** $\{0\}$ **74.** $\left\{\frac{1}{5}\right\}$ **75.** $\left\{-\frac{7}{4}\right\}$ **76.** $\{2\}$ **77.** $a = \frac{F}{m}$ **78.** $I = \frac{E}{R}$ **79.** $P = \frac{k}{V}$ **80.** $g = V - k - t$ **81.** $c = \frac{2A - bh}{h}$ **82.** $x = \frac{4y}{3}$ **83.** 64 and 41 **84.** 36 **85.** 45 **86.** \$11,000 at 8%; \$9,000 at 7% **87.** \$12,000 at 12%; \$13,000 at 19%

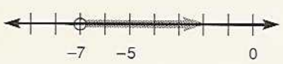
88. $x > 4$;



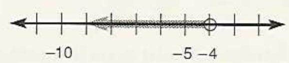
89. $x \le 3$;



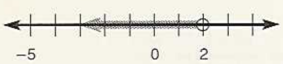
90. $x > -7$;



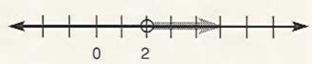
91. $x < -4$;



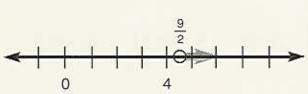
92. $x < 2$;



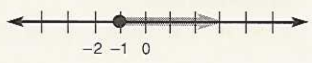
93. $x > \frac{15}{7}$;



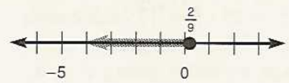
94. $x > \frac{9}{2}$;



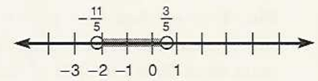
95. $x \ge -\frac{6}{5}$;



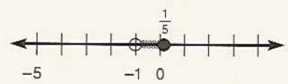
96. $x \le \frac{2}{9}$;



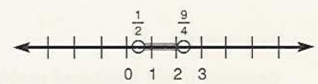
97. $-\frac{11}{5} < x < \frac{3}{5}$;



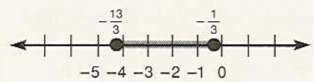
98. $-1 < x \le \frac{1}{5}$;



99. $\frac{1}{2} < x < \frac{9}{4}$;



100. $-\frac{13}{3} \le x \le -\frac{1}{3}$;



Chapter 2 cumulative test

1. -12 **2.** 4 **3.** 3 **4.** undefined **5.** -25 **6.** $\frac{1}{2}$ **7.** $-\frac{1}{6}$ **8.** 0 **9.** 19.78 **10.** 41 **11.** 38 **12.** -24 **13.** 26 **14.** 6 **15.** 20 **16.** $4x$ **17.** $2x^2y^2 + 3xy$ **18.** $a + 3b$ **19.** $4x^2y - 7xy^2$ **20.** $a^3 + 2a^2 + a - 1$ **21.** $-3x + 4y$ **22.** $4a + 2b$ **23.** 64 **24.** -25 **25.** 234 **26.** 2 **27.** 4 **28.** 42 **29.** 51 mph **30.** $x - y$ **31.** (let $x =$ the number) $x + 6$ **32.** $10d + 5n + c$ **33.** $\left\{\frac{5}{3}\right\}$ **34.** $\{0\}$ **35.** $\left\{-\frac{19}{2}\right\}$ **36.** $\left\{\frac{17}{16}\right\}$ **37.** $\left\{\frac{30}{11}\right\}$ **38.** $x \le -6$ **39.** $x < -7$ **40.** $x > \frac{8}{5}$ **41.** $-2 < x < 4$ **42.** $-1 \le x \le 6$ **43.** $b = P - a - c$ **44.** $y = \frac{x - az}{a}$ **45.** \$6,000 at 6%; \$4,000 at 5% **46.** 14,16,18 **47.** 12 **48.** \$6,500 at 12% profit; \$10,500 at 19% loss **49.** $x \le 6$

Chapter 3

Exercise 3–1

Answers to odd-numbered problems

1. a^5 **3.** $(-2)^4$ **5.** x^6 **7.** $(xy)^4$ **9.** $(x - y)^3$ **11.** $xxxx$ **13.** $(-2)(-2)(-2)$ **15.** $5 \cdot 5 \cdot 5$ **17.** $(4y)(4y)(4y)(4y)$ **19.** $(x - y)(x - y)$ **21.** x^{11} **23.** R^3 **25.** a^9 **27.** $5^5 = 3{,}125$ **29.** $4^7 = 16{,}384$ **31.** $(x - 2y)^{10}$ **33.** $(a - b)^{11}$ **35.** x^4y^4 **37.** $64x^3y^3z^3$ **39.** x^{15} **41.** b^{25} **43.** $6x^4y^3$ **45.** a^7b^5 **47.** $30x^5$ **49.** $12a^7b^7c$ **51.** $12a^5b^3$ **53.** $-6a^3b^5$ **55. a.** $V = 5^3$, 125 cubic units **b.** $V = 4^3$, 64 cubic units **c.** $V = 6^3$, 216 cubic units **57.** $s = \frac{1}{2}gt^2$ **59.** $V = \frac{4}{3}\pi r^3$ **61.** $m^4 - 8$ **63.** $2x^2 - y^3$ **65.** $\frac{p^3}{q^2}$

Solutions to trial exercise problems

23. $R^2 \cdot R = R^2 \cdot R^1 = R^{2+1} = R^3$ **27.** $5^2 \cdot 5^3 = 5^{2+3} = 5^5 = 3{,}125$ **43.** $(2xy^2)(3x^3y) = 2 \cdot 3 \cdot xx^3y^2y = 6x^{1+3}y^{2+1} = 6x^4y^3$
55a. $V = e^3$, then $V = (5)^3 = 5 \cdot 5 \cdot 5 = 25 \cdot 5 = 125$ cubic units
60. The cube of Johnny's age is given as n^3. Since his mother is 6 years more than the cube of Johnny's age, we add 6, giving $n^3 + 6$.

Review exercises

1. $9a$ **2.** $8x$ **3.** $6ab$ **4.** $7xy$ **5.** $7a^2 + 5a$ **6.** $5x^2 + 5x$
7. $x^2y + 7xy^2$ **8.** $3ab^2 + 2a^2b$

Exercise 3–2

Answers to odd-numbered problems

1. $2a^3b - 2ab^2c + 2abc^2$ **3.** $15ab^2 - 21ac^2$ **5.** $-15a^3b^2 + 5a^2b^3 - 20ab^4$ **7.** $3a^3b - 6a^2b^2 - 3ab^3$ **9.** $12a^2b^2 - 6ab^3$
11. $x^2 + 7x + 12$ **13.** $y^2 - 13y + 36$ **15.** $a^2 + 2a + 1$
17. $R^2 - 6R + 9$ **19.** $a^2 - 9$ **21.** $6a^2 - 31a + 35$
23. $4x^2 - 49$ **25.** $3k^2 - 17kw - 6w^2$ **27.** $a^2 + 12ab + 36b^2$
29. $4a^2 - 9b^2$ **31.** $a^3 + 2a^2b - 7ab^2 + 4b^3$
33. $6x^3 + 21x^2 - 5x + 28$ **35.** $x^4 - x^3 - x^2 - 11x - 12$
37. $a^3 - 7a^2 + 4a + 12$ **39.** $2a^3 - 3a^2b - 2ab^2 + 3b^3$
41. $a^3 - 3a^2b + 3ab^2 - b^3$ **43.** $a^3 - 6a^2b + 12ab^2 - 8b^3$
45. $cx^2 - 4c^2x + 4c^3$

Solutions to trial exercise problems

8. $(2x)(x - y + 5)(5y) = [(2x)(x - y + 5)](5y) = [2x \cdot x - 2x \cdot y + 2x \cdot 5](5y) = [2x^2 - 2xy + 10x](5y) = 2x^2 \cdot 5y - 2xy \cdot 5y + 10x \cdot 5y = 10x^2y - 10xy^2 + 50xy$
17. $(R - 3)^2$ is a special product. $(R - 3)^2 = (R)^2 + [2 \cdot R \cdot (-3)] + (-3)^2 = R^2 - 6R + 9$
18. $(R + 2)(R - 2)$ is a special product. $(R + 2)(R - 2) = (R)^2 - (2)^2 = R^2 - 4$ **31.** $(a + 4b)(a^2 - 2ab + b^2) = a^3 - 2a^2b + ab^2 + 4a^2b - 8ab^2 + 4b^3 = a^3 + 2a^2b - 7ab^2 + 4b^3$ **37.** $(a - 6)(a - 2)(a + 1) = [(a - 6)(a - 2)](a + 1) = [a^2 - 2a - 6a + 12](a + 1) = [a^2 - 8a + 12](a + 1) = a^3 + a^2 - 8a^2 - 8a + 12a + 12 = a^3 - 7a^2 + 4a + 12$
40. $(a + b)^3 = (a + b)(a + b)(a + b) = [(a + b)(a + b)](a + b) = [a^2 + ab + ab + b^2](a + b) = [a^2 + 2ab + b^2](a + b) = a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Review exercises

1. -5 **2.** 12 **3.** 3 **4.** -9 **5.** x^{12} **6.** a^{15} **7.** $27a^3b^3$
8. $8x^3$

Exercise 3–3

Answers to odd-numbered problems

1. 1 **3.** 5 **5.** 1 **7.** $\frac{1}{R^5}$ **9.** $\frac{1}{9P^2}$ **11.** $\frac{9}{C^4}$ **13.** $\frac{y^3}{2}$
15. $\frac{2y^2}{x^4}$ **17.** $\frac{t^5}{r^2}$ **19.** $\frac{a^6}{b^6}$ **21.** $\frac{8}{27}$ **23.** $\frac{16x^4}{y^4}$ **25.** $\frac{27a^3}{b^3}$
27. y^2 **29.** b^2 **31.** $\frac{1}{R^4}$ **33.** 4 **35.** $\frac{1}{36}$ **37.** y^4 **39.** a
41. $\frac{1}{y^6}$ **43.** a^3b^3 **45.** $4x^2y^2$ **47.** $\frac{1}{27a^3}$ **49.** x^3 **51.** $\frac{1}{a^6}$
53. x^2 **55.** $\frac{1}{a^2}$ **57.** $-\frac{1}{125}$ **59.** 27 **61.** $\frac{1}{a^6}$ **63.** $\frac{1}{x^{10}}$
65. a^6 **67.** 1 **69.** 1 **71.** $\frac{R^5}{S^9}$ **73.** $\frac{ab^4c^2}{8}$

Solutions to trial exercise problems

3. $5a^0 = 5 \cdot 1 = 5$ **10.** $4z^{-2} = 4 \cdot \frac{1}{z^2} = \frac{4}{z^2}$
15. $2x^{-4}y^2 = 2 \cdot \frac{1}{x^4} \cdot y^2 = \frac{2y^2}{x^4}$ **32.** $\frac{3^4}{3^2} = 3^{4-2} = 3^2 = 9$
36. $\frac{x^4x^3}{x^2} = \frac{x^{4+3}}{x^2} = \frac{x^7}{x^2} = x^{7-2} = x^5$ **48.** $\frac{5^2a^3b}{5^3a^7b^3} = 5^{2-3}a^{3-7}b^{1-3} = 5^{-1}a^{-4}b^{-2} = \frac{1}{5^1} \cdot \frac{1}{a^4} \cdot \frac{1}{b^2} = \frac{1}{5a^4b^2}$
49. $x^{-4}x^7 = x^{-4+7} = x^3$ *Alternate:* $x^{-4}x^7 = \frac{1}{x^4} \cdot x^7 = \frac{x^7}{x^4} = x^{7-4} = x^3$ **57.** $(-5)^{-3} = \frac{1}{(-5)^3} = -\frac{1}{125}$ (*Note:* The sign of the base, -5, is unchanged.) **73.** $\frac{4^{-1}a^{-2}b^3c^0}{2a^{-3}b^{-1}c^{-2}} = \frac{\frac{1}{4^1} \cdot \frac{1}{a^2} \cdot b^3 \cdot 1}{2 \cdot \frac{1}{a^3} \cdot \frac{1}{b^1} \cdot \frac{1}{c^2}} = \frac{\frac{b^3}{4a^2}}{\frac{2}{a^3bc^2}} = \frac{b^3}{4a^2} \cdot \frac{a^3bc^2}{2} = \frac{a^3b^{3+1}c^2}{2 \cdot 4a^2} = \frac{a^3b^4c^2}{8a^2} = \frac{b^4c^2}{8} \cdot a^{3-2} = \frac{b^4c^2}{8} \cdot a^1 = \frac{ab^4c^2}{8}$

Review exercises

1. -10 **2.** 21 **3.** 4 **4.** -16 **5.** a^8
6. x^{12} **7.** x^6 **8.** $4a^2b^2$

Exercise 3–4

Answers to odd-numbered problems

1. $8a^6$ **3.** $8x^6y^3$ **5.** $x^{16}y^{12}z^4$ **7.** $125a^{15}b^6c^3$
9. $\frac{1}{4a^4}$ **11.** $\frac{16}{x^4}$ **13.** $\frac{y^{12}}{27x^3}$ **15.** $\frac{y^{10}}{x^4z^6}$
17. $6x^7y^3$ **19.** $-6x^{10}y^4$ **21.** $6x^6y^3z^7$ **23.** $\frac{8x^3}{y^6}$
25. $\frac{9a^4}{b^6}$ **27.** $\frac{8x^6}{y^9}$ **29.** $\frac{4x^6y^6}{z^{10}}$ **31.** $\frac{b^8}{a^5}$
33. $\frac{R^2}{3S^4}$ **35.** $\frac{4}{ab^9}$ **37.** $\frac{3S^3}{R^4}$ **39.** $a^{10}b^{17}$
41. $32a^{12}$ **43.** $32x^{14}y^{13}$ **45.** $\frac{y^2}{xz^4}$
47. $\frac{a^6b^{10}}{4}$ **49.** $\frac{b^6}{a^3c^3}$

Solutions to trial exercise problems

9. $(2a^2)^{-2} = \frac{1}{(2a^2)^2} = \frac{1}{2^2(a^2)^2} = \frac{1}{2^2a^4} = \frac{1}{4a^4}$
31. $\frac{a^{-2}b^3}{a^3b^{-5}} = \frac{\frac{1}{a^2} \cdot b^3}{a^3 \cdot \frac{1}{b^5}} = \frac{\frac{b^3}{a^2}}{\frac{a^3}{b^5}} = \frac{b^3}{a^2} \cdot \frac{b^5}{a^3} = \frac{b^{3+5}}{a^{2+3}} = \frac{b^8}{a^5}$
39. $(a^2b^3)^3(ab^2)^4 = (a^2)^3(b^3)^3 \cdot a^4(b^2)^4 = a^6b^9a^4b^8 = a^{6+4}b^{9+8} = a^{10}b^{17}$

Review exercises

1. 35.34 **2.** 10.36 **3.** 16.72 **4.** 28.98 **5.** 18.81 **6.** 49.5

Exercise 3-5

Answers to odd-numbered problems

1. 2.55×10^2 **3.** 1.2345×10^4 **5.** 1.55×10^5 **7.** 8.55076×10^2 **9.** 1.0076×10^6 **11.** 1.2×10^{-4} **13.** 8.1×10^{-6} **15.** 7×10^{-4} **17.** 9.4×10^{-11} **19.** -4.5×10^3 **21.** -5.85×10^6 **23.** -4.578×10 **25.** -2.985×10^{-8} **27.** 49,900,000 **29.** 7.23 **31.** 0.0042 **33.** 0.00000147 **35.** 0.000789 **37.** −0.00000000482 **39.** −0.00000492 **41.** 1×10^{-9} **43.** 2×10^{12} **45.** 35,600,000 **47.** 0.000 000 000 000 000 000 000 093 **49.** 140,000 **51.** 1.22304×10^{14} **53.** 3.63226×10^{-1} **55.** 1.76979×10^{-7} **57.** 4.84481×10^8 **59.** 1.4×10^3 **61.** 4.6×10^3

Solution to trial exercise problem

58. $(177{,}000) \div (0.15) = \dfrac{1.77 \times 10^5}{1.5 \times 10^{-1}}$

$= \dfrac{1.77}{1.5} \times 10^{5-(-1)} = \dfrac{1.77}{1.5} \times 10^6 = 1.18 \times 10^6$

Review exercises

1. $5x$ **2.** $3a^2$ **3.** $12ab$ **4.** $x^3 + 2x^2$ **5.** $6a^2 - 15a$ **6.** $3x^3y + 2x^2y^2 - 7x^2y$

Chapter 3 review

1. a^{12} **2.** a^{14} **3.** $4^5 = 1{,}024$ **4.** x^4y^4 **5.** a^{15} **6.** $20a^4b^5$ **7.** $6x^3y^7$ **8.** $-15x^5$ **9.** $6a^3b^5$ **10.** $10x^5y^5$ **11.** $-6a^6b^{10}$ **12.** $15x^2 - 10xy$ **13.** $-6a^4b + 9a^3b^2 - 12a^2b^3$ **14.** $30x^2y^2 - 10xy^3$ **15.** $x^2 - x - 12$ **16.** $x^2 + 10x + 25$ **17.** $a^2 - 49$ **18.** $15x^2 + 7xy - 2y^2$ **19.** $x^3 + x^2y - 5xy^2 - 2y^3$ **20.** $3a^3 - 4a^2b - 5ab^2 + 2b^3$ **21.** $8a^3 + 12a^2b + 6ab^2 + b^3$ **22.** $\dfrac{1}{b^2}$ **23.** $\dfrac{5}{a^2}$ **24.** a^4 **25.** $\dfrac{1}{x^3}$ **26.** 1 **27.** $\dfrac{5}{x^3y^2}$ **28.** a^3 **29.** $\dfrac{a^5}{b^5}$ **30.** $\dfrac{4y^2z^2}{x^2}$ **31.** $\dfrac{b^3}{4a^3}$ **32.** $\dfrac{a^9}{b^3}$ **33.** $8a^6b^9$ **34.** $3^{12}x^{16}y^{20} = 531{,}441x^{16}y^{20}$ **35.** $\dfrac{y^6}{4x^2}$ **36.** $\dfrac{xy^3z^3}{2}$ **37.** $\dfrac{2a^2c^3}{b^6}$ **38.** $8x^{26}y^{25}$ **39.** $72a^{14}b^3$ **40.** $\dfrac{a^9b^3}{c^{12}}$ **41.** $\dfrac{9a^6b^4}{c^{10}}$ **42.** $\dfrac{32x^5y^{20}}{z^{30}}$ **43.** 1.84×10^3 **44.** 1.57×10^{-3} **45.** 1.07×10^8 **46.** 8.49×10^{11} **47.** -3.75×10 **48.** -5.43×10^{-3} **49.** 504,000 **50.** 0.00639 **51.** −596 **52.** −0.00886 **53.** 0.000000735 **54.** 812,000,000 **55.** 2.67672×10^5 **56.** 1.54818×10^{-8} **57.** 7.2×10^{-3} **58.** 1.25×10^{-7}

Chapter 3 cumulative test

1. false **2.** false **3.** true **4.** −2 **5.** undefined **6.** 8 **7.** 0 **8.** 16 **9.** 38 **10.** $x^2y + 2xy^2 - 3x^2y^2$ **11.** −40 **12.** 0 **13.** $9x^2 - 6xy + y^2$ **14.** 0 **15.** $4a^4b^{10}$ **16.** −25 **17.** 0 **18.** $9x^2 - 4y^2$ **19.** $6x^3y^5$ **20.** 26 **21.** undefined **22.** $4a + 2b$ **23.** $x^3 - 2x - 1$ **24.** x^2 **25.** a^4 **26.** $\{7\}$ **27.** $\left\{\dfrac{5}{3}\right\}$ **28.** $\left\{-\dfrac{19}{4}\right\}$ **29.** $x > -\dfrac{1}{3}$ **30.** $x > \dfrac{1}{6}$ **31.** $\left\{-\dfrac{3}{2}\right\}$ **32.** $-8 \le x \le -1$ **33.** 97 **34.** 72 **35.** \$18,500 at 8%; \$11,500 at 7%

Chapter 4

Exercise 4-1

Answers to odd-numbered problems

1. $2(y + 3)$ **3.** $4(x^2 + 2y)$ **5.** $3(x^2y + 5z)$ **7.** $7(a - 2b + 3c)$ **9.** $3(5xy - 6z + x^2)$ **11.** $7(6xy - 3y^2 + 1)$ **13.** $2(4x - 5y + 6z - 9w)$ **15.** $5ab(4a - 12 + 9b)$ **17.** $3xy(x + 2)$ **19.** $2R^2(R^2 - 3)$ **21.** $x(2x^2 - x + 1)$ **23.** $3ab(5 + 6b - a)$ **25.** $xy(y + z + 1)$ **27.** $2L(L^2 - 9 + L)$ **29.** $5p(p + 2 + 3p^2)$ **31.** $-3(2x + 3)$ **33.** $2(3x - 4z - 6w)$ **35.** $-3(4L - 5W + 2H)$ **37.** $-x(1 - x + x^2)$ **39.** $-xyz(1 - x + y - z)$ **41.** $-5ab(2ab - 3 + 4a^2b^2)$ **43.** $(a + b)(x + y)$ **45.** $5(2a + b)(3x + 2y)$ **47.** $3(a + 4b)(x + 2y)$ **49.** $(b + 6)(8a - 1)$ **51.** $(a + b)(c + d)$ **53.** $(2a + b)(3x - 2y)$ **55.** $(2x + y)(2a - b)$ **57.** $(5x - 3y)(4x + z)$ **59.** $(a + 3b)(4x - 3y)$ **61.** $(2c - y)(a + 3b)$ **63.** $(c + 4y)(2a + 3b)$ **65.** $(3x + y)(2a + b)$ **67.** $(x - 2d)(3a + b)$ **69.** $(a + 5)(2a^2 + 3)$ **71.** $(4a^2 + 3)(2a - 1)$ **73.** $\pi r(s + r)$ **75.** $\dfrac{wx}{48EI}(2x^3 - 3\ell x^2 - \ell^3)$

Solutions to trial exercise problems

11. $42xy - 21y^2 + 7 = 7 \cdot 6xy - 7 \cdot 3y^2 + 7 \cdot 1$ $= 7(6xy - 3y^2 + 1)$ **25.** $xy^2 + xyz + xy = xy \cdot y + xy \cdot z + xy \cdot 1 = xy(y + z + 1)$ **33.** $6x - 8z - 12w = 2(\quad);$ $2 \cdot 3x - 2 \cdot 4z - 2 \cdot 6w = 2(3x - 4z - 6w)$ **34.** $-4a^3 - 36ab + 16ab^2 - 24b^3 = -4(\quad);$ $(-4)a^3 + (-4)(9ab) + (-4)(-4ab^2) + (-4)(6b^3)$ $= -4(a^3 + 9ab - 4ab^2 + 6b^3)$ **36.** $-3a + a^3b = -a(\quad);$ $(-a) \cdot 3 + (-a)(-a^2b) = -a(3 - a^2b)$ **45.** $15x(2a + b) + 10y(2a + b) = 5(2a + b) \cdot 3x + 5(2a + b) \cdot 2y$ $= 5(2a + b)(3x + 2y)$ **53.** $6ax - 2by + 3bx - 4ay$ $= 6ax + 3bx - 4ay - 2by = (6ax + 3bx) + (-4ay - 2by)$ $= 3x(2a + b) - 2y(2a + b) = (2a + b)(3x - 2y)$

75. $Y = \dfrac{2wx^4}{48EI} - \dfrac{3\ell wx^3}{48EI} - \dfrac{\ell^3 wx}{48EI} = \dfrac{wx}{48EI} \cdot 2x^3 - \dfrac{wx}{48EI} - 3\ell x^2$ $- \dfrac{wx}{48EI} \cdot \ell^3 = \dfrac{wx}{48EI}(2x^3 - 3\ell x^2 - \ell^3)$

Review exercises

1. 4, 5 **2.** 3, 4 **3.** −8, 2 **4.** 8, −2 **5.** 8, 2 **6.** −8, −2 **7.** 6, 6 **8.** 11, 1

Exercise 4-2

Answers to odd-numbered problems

1. $(a + 6)(a + 3)$ **3.** $(x + 12)(x - 1)$ **5.** $(y + 15)(y - 2)$ **7.** $(x - 12)(x - 2)$ **9.** $(a + 8)(a - 3)$ **11.** $(x + 6)(x + 2)$ **13.** $(a - 6)(a + 4)$ **15.** $2(x + 5)(x - 2)$ **17.** $3(x - 8)(x + 2)$ **19.** will not factor, prime polynomial **21.** $(y + 15)(y + 2)$ **23.** $4(x + 2)(x - 3)$ **25.** $5(a - 5)(a + 2)$ **27.** $(xy - 6)(xy + 3)$ **29.** $(xy + 12)(xy + 1)$ **31.** $3(xy + 3)(xy - 4)$ **33.** $(x + 2y)(x + y)$ **35.** $(a - 3b)(a + b)$ **37.** $(a - 3b)(a + 2b)$ **39.** $(x + 3y)(x - 5y)$

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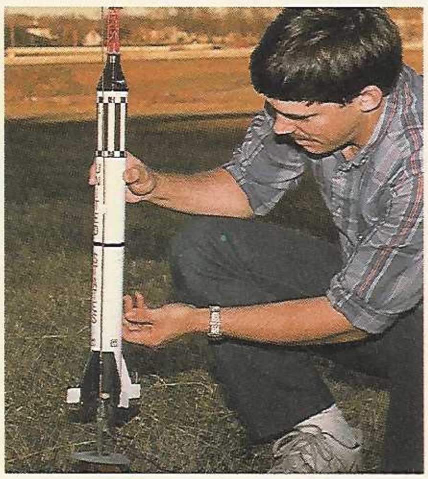

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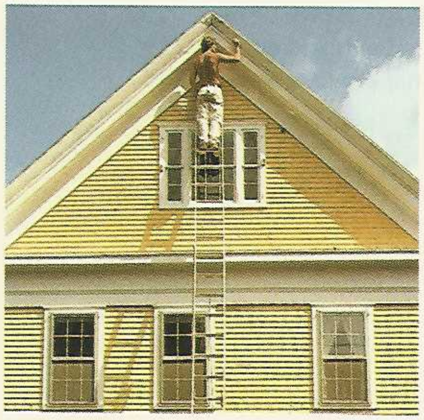

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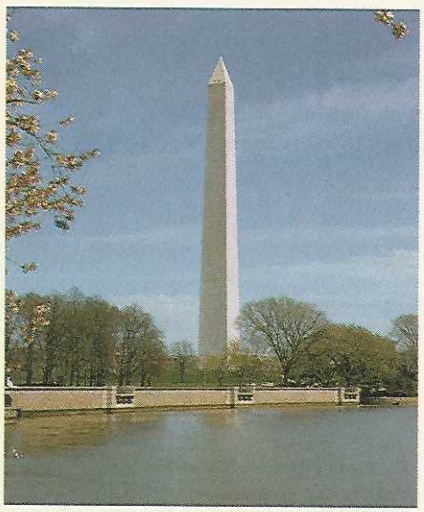

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